Mathematics-High School Conceptual Categories
Number & Quantity

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+), as in this example:

> (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).

All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.

The high school standards are listed in conceptual categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability
- Contemporary Mathematics (Arizona addition)

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

NGA Center for Best Practices (NGA Center), Council of Chief State School Officers (CCSSO). (2012). *Core Standards for Mathematics: Mathematics Standards for High School.* Retrieved August 31, 2012, from Common Core State Standards Initiative: http://corestandards.org/

Mathematics-High School Conceptual Categories
Number & Quantity Introduction

Numbers and Number Systems

During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, "number" means "counting number": 1, 2, 3, … Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

This ascent through number systems makes it fair to ask: what does the word number mean that it can mean all of these things? One possible answer is that a number is something that can be used to do mathematics: calculate, solve equations, or represent measurements.

Although the notion of number changes, the four operations stay the same in important ways. The commutative, associative, and distributive properties extend the properties of operations to the integers, rational numbers, real numbers, and complex numbers. Extending the properties of exponents leads to new and productive notation; for example, since the properties of exponents suggest that $(5^{1/3})^3 = 5^{(1/3)\cdot 3} = 5^1 = 5$, we define $5^{1/3}$ to be the cube root of 5.

Calculators are useful in this strand to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities

In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g. acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process might be called quantification. Quantification is important for science, as when surface area suddenly "stands out" as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

NGA Center for Best Practices (NGA Center), Council of Chief State School Officers (CCSSO). (2012). *Mathematics: High School: Number & Quantity: Introduction.* Retrieved August 30, 2012, from Common Core State Standards Initiative: http://corestandards.org/

Mathematics – Number & Quantity

The Real Number System (N-RN)

Extend the properties of exponents to rational exponents

- **N-RN.1.** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*
- **N-RN.2.** Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers

- **N-RN.3.** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

NGA Center for Best Practices (NGA Center), Council of Chief State School Officers (CCSSO). (2012). *Mathematics: High School: Number & Quantity: The Real Number System.* Retrieved August 30, 2012, from Common Core State Standards Initiative: http://corestandards.org/

Quantities (N-Q)

Reason quantitatively and use units to solve problems

- **N-Q.1.** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- **N-Q.2.** Define appropriate quantities for the purpose of descriptive modeling.
- **N-Q.3.** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

NGA Center for Best Practices (NGA Center), Council of Chief State School Officers (CCSSO). (2012). *Mathematics: High School: Number & Quantity: Quantities.* Retrieved August 30, 2012, from Common Core State Standards Initiative: http://corestandards.org/

The Complex Number System (N-CN)

Perform arithmetic operations with complex numbers

- **N-CN.1.** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
- **N-CN.2.** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
- **N-CN.3.** (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane

- **N-CN.4.** (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
- **N-CN.5.** (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\,i)^3 = 8$ *because* $(-1 + \sqrt{3}\,i)$ *has modulus 2 and argument 120°.*
- **N-CN.6.** (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations

- **N-CN.7.** Solve quadratic equations with real coefficients that have complex solutions.
- **N-CN.8.** (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.
- **N-CN.9.** (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

NGA Center for Best Practices (NGA Center), Council of Chief State School Officers (CCSSO). (2012). *Mathematics: High School: Number & Quantity: The Complex Number System.* Retrieved August 30, 2012, from Common Core State Standards Initiative: http://corestandards.org/

Vector & Matrix Quantities (N-VM)

Represent and model with vector quantities

- **N-VM.1.** (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, *v*).
- **N-VM.2.** (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- **N-VM.3.** (+) Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors

- **N-VM.4.** (+) Add and subtract vectors.
  - Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
  - Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
  - Understand vector subtraction ***v*** – ***w*** as ***v*** + (–***w***), where –***w*** is the additive inverse of ***w***, with the same magnitude as ***w*** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
- **N-VM.5.** (+) Multiply a vector by a scalar.
  - Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
  - Compute the magnitude of a scalar multiple *c**v*** using ||*c**v***|| = |*c*|***v***. Compute the direction of *c**v*** knowing that when |*c*|***v*** ≠ 0, the direction of *c**v*** is either along ***v*** (for *c* > 0) or against ***v*** (for *c* < 0).

Perform operations on matrices and use matrices in applications

- **N-VM.6.** (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
- **N-VM.7.** (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
- **N-VM.8.** (+) Add, subtract, and multiply matrices of appropriate dimensions.
- **N-VM.9.** (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
- **N-VM.10.** (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
- **N-VM.11.** (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
- **N-VM.12.** (+) Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.

NGA Center for Best Practices (NGA Center), Council of Chief State School Officers (CCSSO). (2012). *Mathematics: High School: Number & Quantity: Vector & Matrix Quantities.* Retrieved August 30, 2012, from Common Core State Standards Initiative: http://corestandards.org/